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Research article

MATHEMATICAL MODEL ON POROSITY AND PERMEABILITY INFLUENCE OF PLUG FLOW APPLICATION ON THERMOTOLERANT TRANSPORT IN PHREATIC AQUIFERS

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Abstract

Mathematical model on porosity and permeability influence of plug flow application on Thermotolerant transport in phreatic aquifers has been developed. This model developed is to monitor the behaviour of Thermotolerant at different days and depths, this also including their death rate and their rate of migration with respect to distance at different soils. The derived model generated theoretical values that were compared with experimental results, both parameters compared faviourably, comparison of theoretical values with experimental values from other locations has validate the model as a design tools to applied in design and construction of ground water to prevent water pollution, the results from the theoretical values shows that at aquiferous zone in the study location there is need for treatment, because the deposition of four milligram at thirty metres at hundred days implies that the water at that depth is not good for human consumption, more so the microbes can still survive up to hundred days it implies that if there is substrate deposition for energy, it may increase in microbial population or been inhibited depending on the types of influence that deposit at those depths, the model is imperative, because Thermotolerant is one of major contaminants causing water pollution in the study area.

Keywords: mathematical model, porosity and permeability, plug flow, thermotolerant transport and phreatic aquifers

1. Introduction

The significance of ground water in Water Resources expansion appears not to be attracting the concentration it deserves in Nigeria, despite the fact that a extensive part of Nigeria's Water Supply comes from underground. The quantity of Run-off water or penetration resulting from rainfall is restricted by climatologically and hydrogeological factors which include evapotranspiration, topography vegetation cover, land use, soil moisture content, depth to water table intensity, period and distribution of rainfall etc (Offodile, 2000). While the northern parts of the country fall under the, typically harsh Sahelian Climatic zone, with predominantly high relief underlain by essentially impermeable rocky formations, the southern areas are marked generally, by high humidity with much higher rainfall, low lands and plains underlain by much more permeable sedimentary formations. Each zone is characterized by its peculiar hydrogeological characteristic and ground water potentials, which need to be explored and exploited. The advantage of ground water over surface water lies however, in its availability in virtually every part of the country, though, in varying quantities. In the Sahelian region of the north, surface water is either not available, or is only seasonal. The cost of development of ground water is cheaper than that of surface water and the quality of the water reasonably good requiring only minimal treatment, and in most areas readily potable. Hence it is relevant in the supply of water to the rural and semi urban areas of the country and in some cases even the urban towns, where surface water is not available. The collapse, to intentionally and intensively, incorporate regional ground water exploration and exploitation programs in the overall National Water Resources Development program gives cause for great concern. Ground Water is exploited rather haphazardly and indiscriminately, by government, private institutions, and individuals without any control, management or organization (Offodile, 2000). In the identical method the unit of ground water resources management is the reservoir. In this ground water system, a parallel but adapted water budget is maintained. (Input = Output). Input now implies all forms of recharge into the reservoir or aquifer as infiltration from rainfall, influent river systems, artificial recharge and other forms of recharge, while "Output" indicates discharge from the reservoir, through all forms of abstraction including boreholes wells, springs etc. In ground water basins or reservoirs the attention shifts from a more open system, the river basin or catchment, to a closed up system of the underground reservoir or basin. The geometry structure and disposition of the Nigerian ground water reservoirs, as described *inter alia* (Offodile 1988 Offodile, 2000), determines the ground water potential of any particular area. Nigeria has a coastline that is about 1000km long with the Atlantic Ocean, bordering eight states. These are Lagos, Ogun, Ondo, Delta, Bayelsa, Rivers, Akwa Ibom and Cross River States. While the first four states are west of the River Niger, the last three states are east of the Niger with the last Bayelsa State, straddling the river. Potable water supply to inhabitants in some of the communities in the coastal belt has been a major problem due to salt water intrusion Coastal Nigeria is made up of two sedimentary basins: The Benin basin and the Niger Delta basin separated by the Okitipupa ridge. The rocks of the Benin basin are mainly sands and shales with some limestone which thicken towards the west and the coast as well as down dips to the coast. Recent sediments are underlain by the Coastal Plains Sands which is then underlain by a thick clay layer - the Ilaro Formation and other older Formations (Jones and Hockey, 1964). The Recent Sediments and Coastal Plains Sands consist of alternation of sands and clays. The Recent Sediments forms a water table aquifer which is exploited by hand-dug wells and shallow boreholes. The Coastal Plains Sands aquifer is a multi-aquifer system consisting of three aquifer horizons separated by silty or clayey layers (Longe et al. 1987). It is the main aquifer in Lagos Metropolis that is exploited through boreholes for domestic and industrial water supply. In the coastal belt of the Benin basin, this aquifer is confined. The Niger Delta is a coastal arcuate delta of the River Niger covering an area of about 75,000km^2. The subaerial Niger Delta has an extensive saline/brackish mangrove swamp belt separated from the sea by sand beach ridges for most of the coastline. Water supply problems relating to salinity are confined to the saline mangrove swamp with associated sandy islands and barrier ridges at the coast. Geologically, rocks of the Niger Delta are subdivided into three Formations which are Akata, Agbada and Benin Formations (Short and Stauble, 1967). The Benin Formation consisting predominantly of massive highly porous sands and gravels with locally thin shale/clay interbeds forms a multi-aquifer system in the delta. Many boreholes have been drilled into the aquifers of the Benin Formation yielding good quality water but many have also been abandoned due to high salinity. Oil and gas are produced from sand reservoirs in the Agbada Formation while the Akata Formation consists of uniform shale rocks. In the Benin basin, although groundwater is a renewable resource, fear is being nursed about its imagined danger in case of inadequacy. The universality of its utility heightens the degree of fear as no other fluid can replace the uncountable roles played by water in the life of plants and animals. Groundwater is ubiquitous but sometimes its availability in economic quantity depends solely on the distribution of the subsurface geomaterials which are referred to as the aquifers. This implies that where groundwater is not potentially endowed enough, there may be either complete lack or inadequacy due to increasing industrial and domestic needs (Akpan, et al 2006, George et al 2010). The thickness and the distribution of water bearing geomaterials sometimes do not really predict the potential reserve since some of the delineated aquifer might be ideally non porous, dry and non prolific (Pentelis Ms, Maria, et al 2007). The determination of porosity of the saturated delineated geomaterials is the sure way of estimating the usable capacity of aquifers and the groundwater reserve in a given area (Hago, 2000). This traditional technique measures the geometry of the aquifer and the portion of it that is saturated with groundwater which is now gradually replacing the surface water that is degraded in many ramifications. The study area lies between longitudes 7o451 to 8o101E and latitudes 4o30' to 5o10'N. The geology of the area is Recent to Tertiary Sediments belonging to the Benin Formation. This formation is the youngest geologic unit in the Niger-Delta Sedimentary Basin. This formation comprises continental sand and gravel, deposited on the upper deltaic plain environment. The grain sizes range from coarse to fine sand in textures and are poorly sorted (Reijers et al 2004). They are also thick and friable with minor intercalations of clay, silts and sandstones in the area mapped. The alternative sequence builds up multiple-aquifer systems with various thicknesses (Griffiths and kings, 1985; Mbonu, et al 1997). Thus in the study area, the aquifer systems have been found to be a combination of ensemble of different grain sizes of sand The survey area sits on a relatively flat terrain and is drained actively by the some fresh and saline River. It has humid tropical climate, characterized with two distinct seasons – the wet and dry seasons. Most of the coastal areas in the study area usually have salt-fresh water interface at certain depths. The aquifers are predominantly unconfined to semi-confined and this makes them to be water table aquifers. The static water levels in the area have minor variations and the results are minor hydraulic gradients observed in the area. The general recharge is from the surface flows and from the rivers which surround the study area (Mbonu et al 1997).

2. Materials and Method

Analytical model were developed, applying mathematical tools, the derived model were applied produced theoretical values that were compared with experimental laboratory analysis. The experimental procedure is column experiment, the soil samples were collected at intervals of three metres each (3m). Thermotolerant solute was introduced at the top of the column and effluents from the lower end of the column were collected and analyzed for Thermotolerant that generated results from its analysis.

Developed Mathematical Model

Developed Model of Thermotolerant in Groundwater was developed considering the Thermotolerant growth rate function to be dependent of velocity, time and distance. Based on these conditions as stated above a general mathematical expression can be written as:

Mass balance on plug flow system in soil porosity and permeability can be expressed as

INPUT – OUTPUT – REACTION = ACCUMULATION

$$VoCAf = V \bullet \left[Vo\left(CAf + d\,CAf\right)\right] \bullet \left[\left(Vav\right)\partial Z\right] = \frac{\partial}{\partial t} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$q\partial CAf \bullet \partial Z \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Dividing the equation by *dz* and taking limit *adz* *0* ……………→ (3)

$$\varepsilon = \frac{\partial CAf}{\partial t} + Vo\frac{\partial CAf}{\partial z} + Vav = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

But for first order reaction fluid only

$$YAv\left[\frac{Mol}{M^3\,reactor}\right] - \frac{1}{V}\frac{\partial NA}{\partial z} + Kd(nk)CAf \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

Where,

n = Porosity and

K = permeability of the soil

$$\frac{\partial CAf}{\partial z} = 0$$

Therefore, $$Vo\frac{\partial C}{\partial z} + K(nk)CAf = 0 \qquad \text{(6)}$$

Considering the function of height integrating with $CAf = CAf$ at $Z = 0$

$$X_A = \left[\frac{CAf}{CAf\ in} - \right] - \exp K_1^n\left((nk)Z\right) \qquad \text{(7)}$$

Balance on solid state

- ∂A (Fluid) + (Solid) ⟶ Product
- Input – Output reaction = Accumulation
- Over increment of ∂Z: Input = 0 Output = 0.

$$-\left(YSv - \right)(nk)\frac{\partial C}{\partial t} - \Delta Z \qquad \text{(8)}$$

$\dfrac{\text{Porosity and permeability}}{M^3\ of\ reactor\ volume}$ $\qquad$ $\dfrac{Mol}{M^3\ reactor}$

$$(nk)\frac{\partial C}{\partial t} + YSv = 0 \qquad \text{(9)}$$

$$-(YSv) = \alpha(YSv) \qquad \text{(10)}$$

$$(nk)\frac{\partial C}{\partial t}Af + \frac{\partial CAf}{\partial Z}Af + Y\alpha v = 0 \qquad \text{(11)}$$

Substituting YAf equation (12) yields

$$\frac{\partial Cs}{\partial t} + \frac{Y\alpha v}{\alpha(nk)} = 0 \qquad \text{(12)}$$

$$\frac{\partial Cs}{\partial t} + \frac{Y\alpha v}{\alpha(nk)} - 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

$$\frac{\partial CAf}{\partial Z} - \frac{\partial Cs}{\partial t} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

$$C^1 Af = f(Z,t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

$$C^1 s = f(Z,t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

Thermotolerant transport is a continuous process as reflected in plug system application influenced by porosity and permeability.

Solve

$$\frac{\partial CAf}{\partial Z} - \alpha(nk)\frac{\partial C}{\partial t} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (17)$$

Considering when CAf = Cs, thus equation (17) can be written as

$$\frac{\partial C}{\partial Z} - \frac{(nk)}{Vo}\frac{\partial C}{\partial t} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (18)$$

Where $C = CAf = Cs$ ………………………………………. (19)

Applying separation of variables considering the coordinate of Z in terms of time dependent, thus equation (18) can be expressed as

$$C = TZ$$

Integrating boundary conditions are $t = 0, \quad C = CoZ$

Therefore, $$\frac{\partial C}{\partial Z} = TZ^1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

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$$\frac{\partial C}{\partial Z} = T^1 Z \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (21)$$

Integrating equation (20) and (21) into equation (18) yield

$$TZ^1 \left(\ell^{nk}\right) T^1 Z = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (22)$$

Therefore

$$TZ^1 = \ell^{(nk)} T^1 Z = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (23)$$

$$\frac{Z}{Z} = \ell^{nk} \frac{T^1}{T^1 Z} = \lambda^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (24)$$

$$Z^1 = \lambda^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (25)$$

$$\frac{1}{Z} \frac{\partial Z}{\partial Z} = \lambda^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (26)$$

$$\int \frac{\partial Z}{\partial Z} = \int^{-\lambda^2} \partial Z \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (27)$$

$$Ln\ Z = -\lambda^2 Z + C_1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (28)$$

$$Z\lambda^2 + C_1 = \ell^{-\lambda^2 Z} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (29)$$

$$Z = A_0 - \ell^{\lambda^2 Z} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (30)$$

$$\ell^{(nk)} \frac{T^1}{T} = \lambda^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (31)$$

$$Ln\ T = \frac{\lambda nk}{\alpha} + C_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (32)$$

$$\ell^{-\lambda^2 \frac{nkt}{\alpha}} \bullet \ell^{C_2} \qquad \ldots\ldots \qquad (33)$$

$$T = \beta \ell^{\frac{\lambda^2 nkt}{\alpha}} \qquad \ldots\ldots \qquad (34)$$

But $C = TZ$

$$C = \beta^{\frac{\lambda^2 nkk}{\alpha}} \bullet A\ell^{-\lambda^2 Z} \qquad \ldots\ldots \qquad (35)$$

$$i.e.\ C = A\beta^{\frac{\lambda^2 nk}{\alpha}t} {}^{-Z} \qquad \ldots\ldots \qquad (36)$$

At $0\ Zo = C_{(o)} = Co$

$$C = C_o \ell^{\frac{\lambda^2 (nktZ)}{\alpha}} \qquad \ldots\ldots \qquad (37)$$

Therefore, Transfer in the above equation into sinusoidal curve, so that we have

$$C = C_o Sin\left(\frac{nk}{\alpha}t + Z\right) \qquad \ldots\ldots \qquad (38)$$

$$C = C_o Sin\left(\frac{nk}{\alpha}t + Z\right)V \qquad \ldots\ldots \qquad (39)$$

Table 1: Comparison of theoretical and experimental values at various depths

Depth	Theoretical values conc mg/l	Experimental Values mg/l
3	4.42	4.64
6	0.02	0.05

9	0.31	0.33
12	0.54	0.52
15	2	1.88
18	8.14	8.11
21	14.05	14.77
24	11.7	11.13
27	4.98	5.04
30	4.8	4.65

Table 2: Comparison of theoretical and experimental values at various depths

Time	Theoretical values conc mg/l	Experimental Values mg/l
10	4.42	4.64
20	0.02	0.05
30	0.31	0.33
40	0.54	0.52
50	2	1.88
60	8.14	8.11
70	14.05	14.77
80	11.7	11.13
80	4.98	5.04
100	4.8	4.65

Table 3: Theoretical values at various depths

Time	conc mg/l
10	4.42
20	0.02
30	0.31
40	0.54
50	2
60	8.14
70	14.05
80	11.7
80	4.98
100	4.8

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Table 4: Theoretical values at various depths

Depth	conc mg/l
3	4.42
6	0.02
9	0.31
12	0.54
15	2
18	8.14
21	14.05
24	11.7
27	4.98
30	4.8

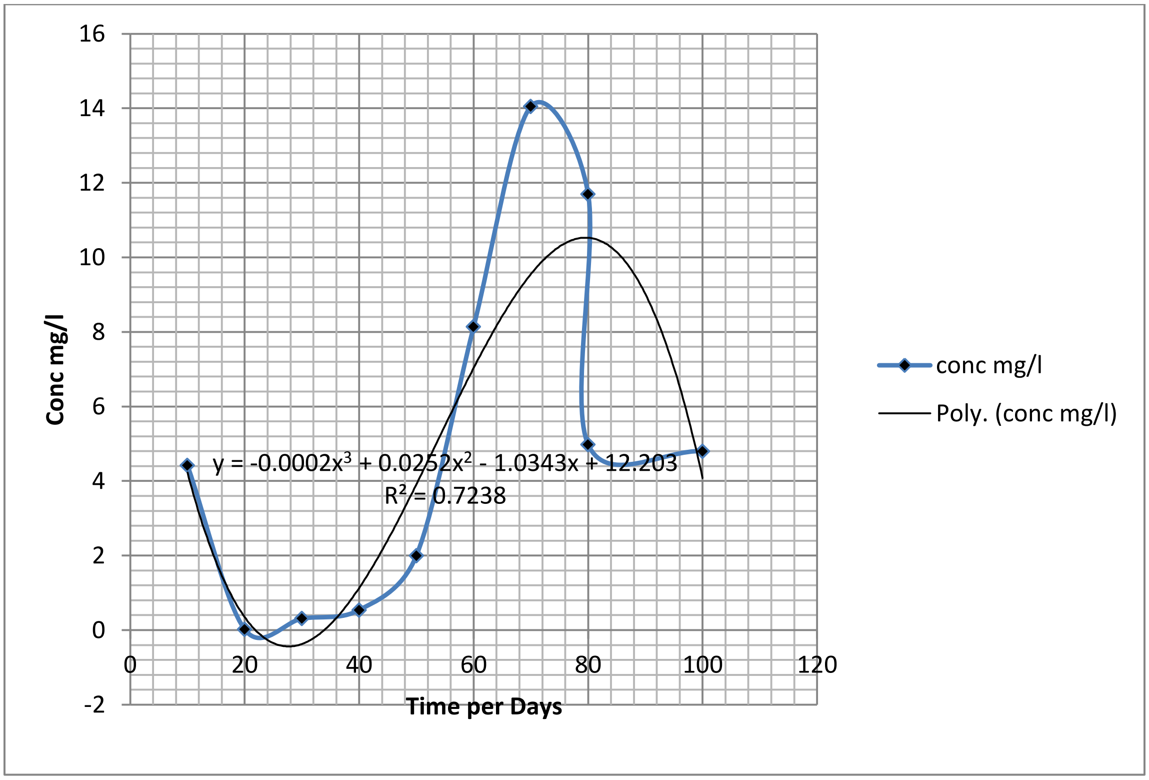

Figure 1: Theoretical and experimental values at various depths

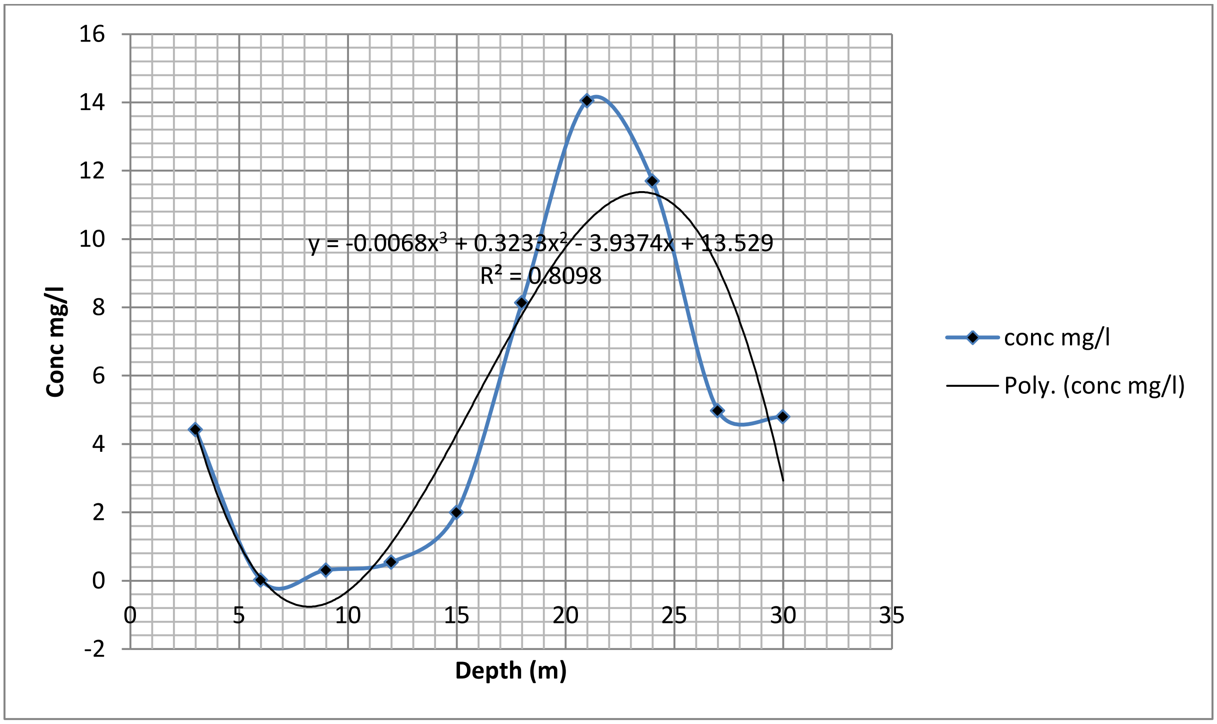

Figure 2: Theoretical and experimental values at various depths

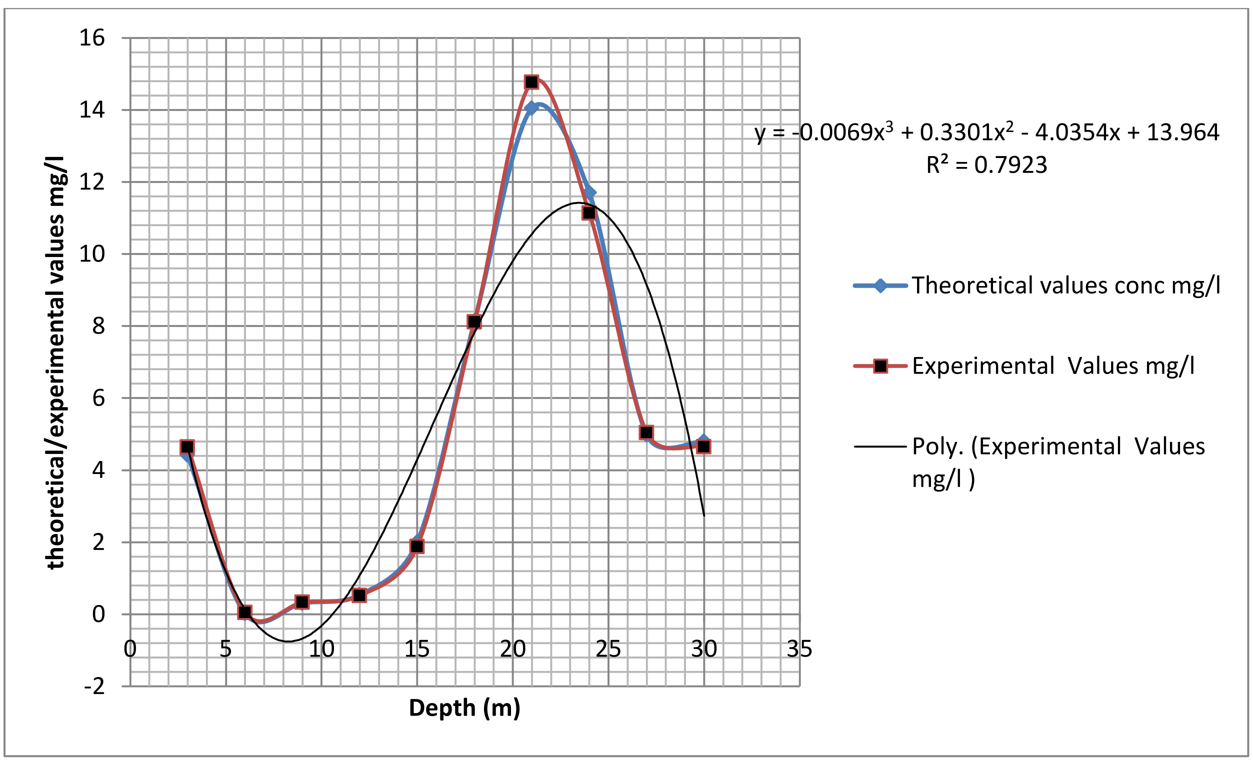

Figure 3: Comparison of theoretical and experimental values at various depths

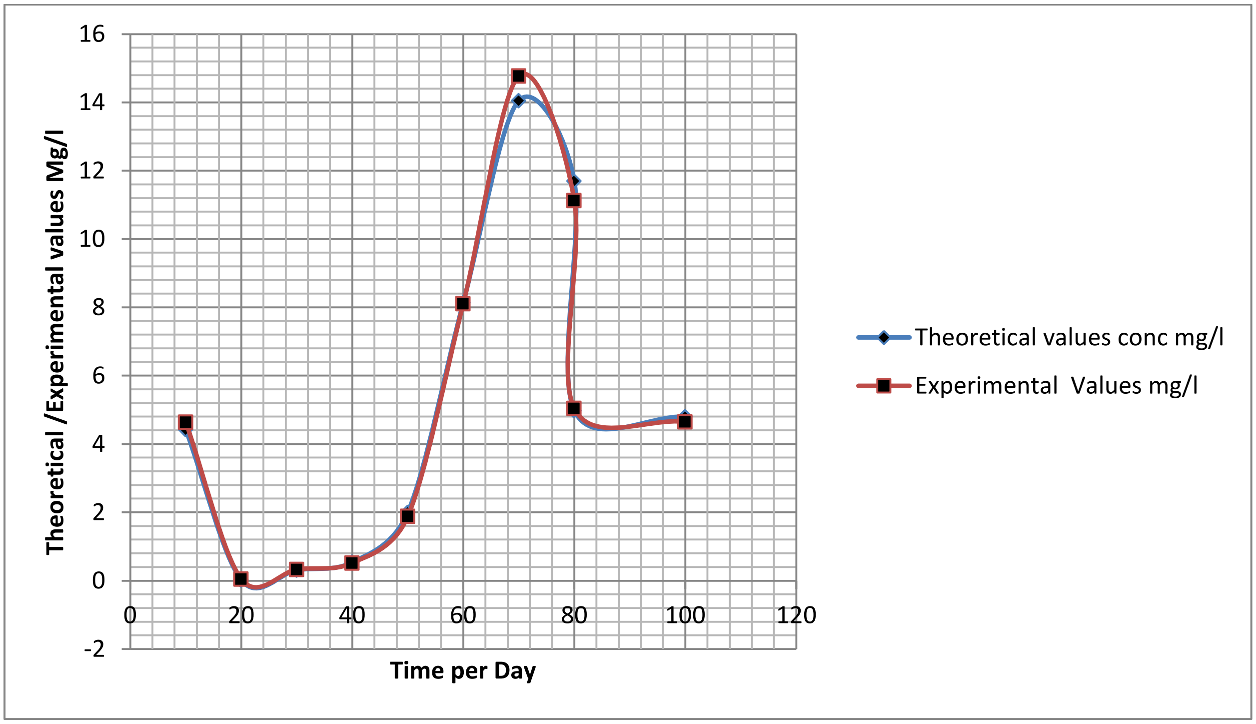

Figure 4: Comparison of theoretical and experimental values at various depths

At homogeneous stratum, the microbes gradually increase to the point where the optimum values were recorded at seventy days, in a similar condition degradation of the microbial concentration was observed from eight to hundred days. Similar condition where observed with respect to distance the microbes gradually increase and record it optimum values at twenty one metres, it suddenly decrease with depth to four milligram per liter at hundred days and thirty metres respectively. The results of both parameters compared shows that there is a contaminants of Thermotolerant concentration at the aquiferous region, this condition implies that ground water at those region need a thorough design or treatment, to prevent the Thermotolerant deposition Comparing the theoretical values with experimental values, the results at various days and at different depths compared faviourably, the experimental results from other locations in the study area compared validated the model, this condition explained that the model developed can be applied as a tool for design and construction of ground water preventing water pollution in the study location. Furthermore, the developed model on permeability and porosity influence on Thermotolerant transport in Phreatic aquifers has explained the behaviour of the species that will definitely vary from other microbes in terms of migration due to reaction with other influence deposited in the formation, the condition in terms of the microbial behaviour has detailed the variation on the influence of migration at different formation, since the depositional influence in the soil structure has a lots of variations that influence the behaviour of different microbes on their transport condition to ground water aquifers.

4. Conclusion

The deposition of Thermotolerant influenced by permeability and porosity has generated some results from the developed model, the experimental results were applied to validate the model, and both parameters compared faviourably. The results from both parameters shows the behaviour of the microbes at different time and depths, the derived model for porosity and permeability has explain the rate of influence on Thermotolerant transport to phreatic aquifers. Results at thirty metres and at hundred days shows the rate of Thermotolerant concentration, furthermore it has explain that at hundred days the microbes are still living and at thirty metres the concentration is still harmful to human health, the microbes generating that values at hundred days at thirty metres also explain that some other depositional influence may generate energy and increase their microbial population or in some instance on there migration, they may also experience inhibition as well as degradation, this condition definitely need factor of safety in design, because further depths or days apart from hundred days at thirty metres, the microbes may experience this two conditions stated. The design must integrate factor of safety to avoid failure so that when constructing water well the detailed condition from the model will be use to prevent abortive well or water pollution.

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